Supplementary Materials

An-Tsun Wei, Hui Wang

# S.I. Figures

**一張含有 圖表 的圖片

自動產生的描述**

Fig. S1. Example of discontinuity detection (vertical green lines) in slopes (red lines).

**A diagram of a wavelet

Description automatically generated**

Fig. S2. Example of change-point detection (red points) via wavelet decomposition.

A black and white square with a blue square with a red and yellow line

Description automatically generated

Fig. S3. Mapping process of printing defects from an image of a single layer through an image segmentation tool in MATLAB.

**一張含有 圖表 的圖片

自動產生的描述**

Fig. S4. Kinematic-induced variation caused by acceleration and deceleration [30].

A screenshot of a computer

Description automatically generated

Fig. S5. The interaction between global and local parameters demonstrated using printing speed.

A diagram of a pattern

Description automatically generated

Fig. S6. Visualization of the process for user printers to fine-tune the learning-useful pattern from the cloud data decomposed and pre-trained using *BMM*.

A graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of

Description automatically generated

Fig. S7. An expected result of global speed optimized in the pre-trained and fine-tuned stage.

A yellow square with arrows and a black background

Description automatically generated

Fig. S8. The printing path from slicing for on single-layer square print.

A white rectangular table with numbers

Description automatically generated

Fig. S9. The data structure for *BLR* training.

A black square with white lines

Description automatically generated with medium confidence

Fig. S10. Maximum speed for acceptable quality through sequential

experimentation in regular printing.

A collage of different shapes

Description automatically generated with medium confidence

Fig. S11. The example of simulated cloud data.

A diagram of a curve

Description automatically generated

**(a)**

A screenshot of a graph

Description automatically generated

**(b)**

Fig. S12. (a) The 94% HDIs derived from MCMC posterior sampling for , and (b) diagnostic analysis of MCMC sampling using multiple chains, focusing on the overlap observed in trace plots from different runs.

A screenshot of a graph

Description automatically generated

Fig. S13. Prediction of defective regions at based on regions learned from two user samples.

# S.II. BMM FOR PRIOR SELECTION

Equations (S1.1-S1.5) formulate the by introducing a latent variable to reveal the number of hidden features within the observed cloud data.

**Mixture model:**

*∀*  (S1.1)

(S1.2)

**Prior features:**  (S1.3)

(S1.4)

(S1.5)

The model employs a categorical distribution for feature assignments , parameterized by probabilities derived from a Dirichlet distribution. The integrates features to model the outcome ​ of each trial as influenced by a distinct mixture of these features. It can be represented as where represent the parameter for the -th feature in , respectively. also indicates the mixing coefficient or contribution of the -th feature within the mixture, with the constraint ensuring a proper probabilistic mixture. For , the posterior of and the model parameters are typically inferred using iterative algorithms like the Expectation-Maximization algorithm or Variational Bayes methods (as research interest). Let represent the mode of the posterior distribution , and represents the superset of all the features. Then, we determine the best combination of features denoted as by minimizing the mean-square error (MSE) to in (S2), where is the binary set converted by , which is the same data structure as . The probability given the cloud data and is calculated as . Hence, the informative cloud prior for modeling the quality of each spacing is based on the same parametrization in (4.2) given . This enables users to accurately learn even without replication.

# S.III. CONSIDERATION OF INPUT-DEPEDENT NOISE VARIANCE IN GAUSSIAN PROCESS

Consider the quality of each spacing that follows (i.e., ), the count of effective compensations in sequential Bernoulli trials will follow a Binomial distribution. This allows *GP* to consider the variance in printing duration influenced by the count of effective compensations regarding spacing. The variance can be converted into time unit basis as , where is the scale factor determined by the time difference between successfully compensated on spacing versus those that are not, influenced by specific local parameters. In a standard , is set to 1, reflecting outcomes where success is coded as 1 and failure as 0. Adjusting to reflect printing time, it's determined based on the discrepancy between the fastest print time for no defect and the minimum required time for printing a defect in single spacing. For simplicity in this paper, we set as a time unit without prior knowledge. To scale the variance relative to the fastest uncompensated printing time and ensure consistent scaling while maintaining outcomes within the range from 0 to 1, the ratio of variance is derived as , which can be considered as the input-dependent noise variance in the pre-trained *GP* model. For a large number of trials, according to the Central Limit Theorem, the variance of the binomial distribution can approximate that of a Gaussian distribution, enabling the incorporation of binomial-derived variance into the input noise variance of *GP*.

# S.IV. PARAMETER SETUP IN DETAIL

For the irregularity detection, we set the constraint of the minimum distance of ten s between two irregularities in . To further construct the area , are determined at 5 for grouping the nearest selected spacings for compensation at based on ,.

Since the printing speed is the only kinematics parameter we used for the error compensation in G-code, and the prior distribution can be constructed in (S3.1-S3.3).

(S3.1)

(S3.2)

(S3.3)

are set as a large value at 10. Based on the process knowledge of the printing experiment, we know that the probability of getting a good quality might have an inverse correlation to speed and . The higher speed setting and , the lower the chance of getting good quality. However, we can construct the informative prior by leveraging the process knowledge on , using Truncated Normal distribution to constrained , can only be negative. After considering the data , MCMC is used for sampling the posterior distributions for , , and generate the 94% HDIs for each (refers to (8.2)) between 3% and 97% in Fig. S12(a). The MCMC diagnostics (Fig. S12(b)) is also provided for testing the stationarity of the posterior sampling by inspecting the overlap between chains in different runs.

After obtaining the 94% HDIs of the *BLR* model coefficients, the two-stage optimization (8.1-9.3) can be implemented. Since speed is the only kinematics parameter for the G-code compensation, according to the optimized output (i.e., ) from the first optimization, the second optimization for can be simplified by the calculation in (S4) given .

(S4)

# S.V. SIMULATED PRE-TRAINED PRINTERS

The Gaussian distribution accounts for the cluster's central tendency and dispersion, which is used to reflect how printing speed affects the size of the defective area centered around the pre-defined irregularity. The mathematical model in (S5) allows us to simulate how the probability of defects occurring is distributed across the infill spacings. Specifically, as the printing speed increases, the denominator of the exponential function increases, leading to a wider spread of spacings around with high probability. Conversely, slower speeds result in more concentrated regions with high probability.

(S5)

selected from 𝕄 specifies the mean position along the infill pattern where the defects are most concentrated, simulating the defective spacings. determines the dispersion around each . represents the relative intensity or frequency of defects associated with each cluster within the infill pattern. is a hyperparameter that controls the variance of the cluster across the infill pattern, effectively tuning the sensitivity of defects spread to changes in printing speed. The parameters of the simulated pre-trained printer in this case study are set as

# S.VI. COMPENSATION RESULTS FOR GLOBAL & LOCAL PARMATERS

The left plots in Fig. S13 demonstrate the defect regions constructed by employing an informative cloud prior to the binary infill data at speeds of 150 and 25 mm/s. By fine-tuning the best pre-trained *GP* based on the user ratios, the optimal global speed is derived at 87 mm/s. By observing the infill samples at speeds of 150 and 25 mm/s, the defect areas progressively diminish moving from left to right when speed decreases. This phenomenon allows this study to predict the posterior defect regions at a global speed of 87 mm/s, utilizing linear interpolation as shown in the right plot in Fig. S13. The predicted results show that the number of spacings covered by three defective regions are 24, 64, and 35 respectively, and their local speeds, optimized via the fine-tuned *BLR* are 69, 31, and 57 mm/s respectively. Once the **,** **,** are obtained, this paper validates the two-stage global-local learning process by comparing the printing results with regular printing and the compensation with local optimization under the fastest global speed .